

Extracting Weak Phases Cleanly from Charmless 3-Body B Decays¹

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Abstract

In the past, it was believed that one cannot obtain clean weak-phase information from the measurement of CP-violating asymmetries in 3-body B decays. Recently it was shown that this is not true – by expressing the decay amplitudes in terms of diagrams and using Dalitz plots, one can resolve all the difficulties and cleanly extract weak phases. In this talk I describe how this is done, and present preliminary results on the measurement of γ using the decays $B \rightarrow K\pi\pi$ and $B \rightarrow KK\bar{K}$.

The standard method for obtaining clean information about the weak Cabibbo-Kobayashi-Maskawa (CKM) phases is through the measurement of indirect (mixing-induced) CP-violating asymmetries in $B^0(t) \rightarrow f$. This requires that f be a CP eigenstate. Because of this, the conventional wisdom is that one cannot obtain such clean CKM information from 3-body decays since final states such as $K_S\pi^+\pi^-$ are not CP eigenstates – the value of its CP depends on whether the relative $\pi^+\pi^-$ angular momentum is even (CP +) or odd (CP –).

There are some exceptions. If the final state contains truly identical particles – e.g. $K_S\pi^0\pi^0$ – it is a CP eigenstate. (Here the relative $\pi^0\pi^0$ angular momentum is necessarily even, which means the state is CP +.) Also, for $B^0 \rightarrow K^+K^-K_S$, Belle used an isospin analysis to differentiate CP + and CP –. They found that it is dominantly CP + [1].

Unfortunately, even for these exceptions, there is an additional problem. The procedure for getting clean weak-phase information from indirect CP asymmetries only works if the decay is dominated by amplitudes with a single weak phase. However, in general these decays receive significant contributions from amplitudes with a different

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weak phase. In order to extract the weak phases, one needs a way of dealing with this “pollution.”

Recently it was shown that all of these difficulties can be overcome [2, 3, 4]. I describe the method below.

The first ingredient is the use of Dalitz plots. Consider the decay $B \rightarrow P_1 P_2 P_3$ (P is a pseudoscalar meson), in which each P_i has momenta p_i . One can construct the three Mandelstam variables:

$$s_{12} \equiv (p_1 + p_2)^2, \quad s_{13} \equiv (p_1 + p_3)^2, \quad s_{23} \equiv (p_2 + p_3)^2. \quad (1)$$

These are not independent, but obey the relation

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2. \quad (2)$$

The Dalitz plot is given in terms of two Mandelstam variables, say s_{12} and s_{13} . For the decay amplitude, we write

$$\mathcal{M}(B \rightarrow P_1 P_2 P_3) = \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}). \quad (3)$$

Here the sum is over all decay modes (resonant and non-resonant). c_j and θ_j are the magnitude and phase of the j contribution, relative to one of the channels. The distributions F_j describe the dynamics of the individual decay amplitudes, and take different forms for the various contributions. The key point is the following: in the experimental Dalitz-plot analyses, explicit expressions for the F_j are assumed (e.g. Breit-Wigner). Then a maximum likelihood fit over the entire Dalitz plot gives the best values of the c_j and θ_j . Thus, *the decay amplitude $\mathcal{M}(s_{12}, s_{13})$ is known.*

With this information the CP of the final state can now be fixed. For example, suppose the final state has CP + when the amplitude is symmetric under $P_2 \leftrightarrow P_3$ (as is the case for the final state $K_S \pi^+ \pi^-$). We can find this amplitude from the above:

$$\mathcal{M}_{sym} = \frac{1}{\sqrt{2}} [\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12})]. \quad (4)$$

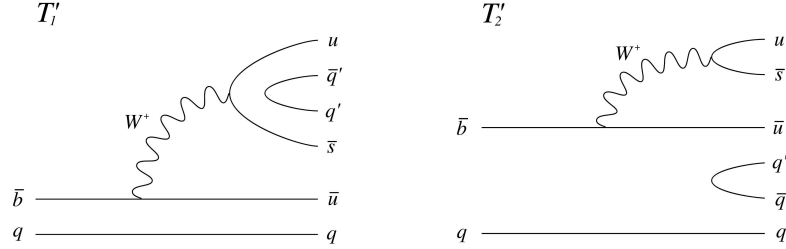
Using this, it is possible to compute the $B \rightarrow P_1 P_2 P_3$ observables. E.g. the indirect CP asymmetry is given by

$$S = \text{Im} \left[e^{-2i\phi_M} \frac{\overline{\mathcal{M}}_{sym}}{\mathcal{M}_{sym}} \right]. \quad (5)$$

Note: all observables are momentum dependent – they take different values at each point in the Dalitz plot.

The second ingredient is the use of diagrams [2]. In 2-body decays all amplitudes are expressed in terms of color-allowed and color-suppressed tree, gluonic penguin,

and electroweak penguin (EWP) diagrams; annihilation/exchange-type diagrams are neglected. In 3-body decays, one has similar diagrams. Here one has to “pop” a quark pair from the vacuum. For the diagrams we add the subscript “1” if the popped quark pair is between two non-spectator final-state quarks, and “2” if it is between two final-state quarks including the spectator.



The above figure shows the T'_1 and T'_2 diagrams contributing to $B \rightarrow K\pi\pi$ (as this is a $\bar{b} \rightarrow \bar{s}$ transition, the diagrams are written with primes). The other diagrams ($C'_1, C'_2, P'_1, P'_2, P'_{EW1}, P'_{EW2}, P'^C_{EW1}, P'^C_{EW2}$) are obtained similarly from the 2-body diagrams.

Note: unlike the 2-body diagrams, the 3-body diagrams are momentum dependent. This must be taken into account whenever the diagrams are used.

Now, in $B \rightarrow K\pi$ decays there are relations between the EWP and tree diagrams under flavor SU(3) symmetry [5]. Recently it was shown that similar EWP-tree relations hold for $B \rightarrow K\pi\pi$ decays [3]. The Wilson coefficients obey $c_1/c_2 = c_9/c_{10}$ to about 5%, in which case these relations take the simple form (the exact relations are given in Ref. [3])

$$P'_{EW1} = \kappa T'_1, \quad P'_{EW2} = \kappa T'_2 \quad ; \quad P'^C_{EW1} = \kappa C'_1, \quad P'^C_{EW2} = \kappa C'_2, \quad (6)$$

where

$$\kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2}, \quad (7)$$

with $\lambda_p^{(s)} = V_{pb}^* V_{ps}$.

However, there is an important caveat. Under SU(3), the final state in $B \rightarrow K\pi\pi$ involves three identical particles, so that the six permutations of these particles (the group S_3) must be taken into account. But the EWP-tree relations hold only for the totally symmetric state. Thus, the analysis must be carried out for this state. The fully symmetric state can be found from the Dalitz plot. Instead of the amplitude which is symmetric only under $P_2 \leftrightarrow P_3$ [Eq. (4)], we define

$$\begin{aligned} \mathcal{M}_{fully \ sym} = & \frac{1}{\sqrt{6}} [\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) + \mathcal{M}(s_{12}, s_{23}) \\ & + \mathcal{M}(s_{23}, s_{12}) + \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23})] . \end{aligned} \quad (8)$$

All observables, such as the indirect CP asymmetry [see Eq. (5)] are computed using $\mathcal{M}_{fully\ sym.}$.

Once the full decay amplitudes are expressed in terms of diagrams, one can perform an analysis like that done with 2-body decays – one can combine the amplitudes for different decays in order to isolate and extract a CKM phase. I now give an example of such an analysis involving $B \rightarrow K\pi\pi$ and $B \rightarrow KK\bar{K}$ decays [4]. (Note: SU(3) is assumed.)

There are 6 decays of the type $B^+/B^0 \rightarrow K\pi\pi$. Decays with two π^0 's are excluded as being too difficult experimentally. Also, $B^+ \rightarrow K^0\pi^+\pi^0$ is not independent – its amplitude is proportional to that of $B^0 \rightarrow K^+\pi^0\pi^-$ [2]. There are therefore only three $B \rightarrow K\pi\pi$ decays to consider.

The $B \rightarrow K\pi\pi$ amplitudes in which the $\pi\pi$ pair is symmetrized are:

$$\begin{aligned} 2A(B^0 \rightarrow K^+\pi^0\pi^-)_{sym} &= T'_1 e^{i\gamma} + C'_2 e^{i\gamma} - \kappa(T'_2 + C'_1) , \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^+\pi^-)_{sym} &= -T'_1 e^{i\gamma} - C'_1 e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} + \tilde{P}'_{tc} \\ &\quad + \kappa\left(\frac{1}{3}T'_1 + \frac{2}{3}C'_1 - \frac{1}{3}C'_2\right) , \\ \sqrt{2}A(B^+ \rightarrow K^+\pi^+\pi^-)_{sym} &= -T'_2 e^{i\gamma} - C'_1 e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} + \tilde{P}'_{tc} \\ &\quad + \kappa\left(\frac{1}{3}T'_1 - \frac{1}{3}C'_1 + \frac{2}{3}C'_2\right) . \end{aligned} \quad (9)$$

These expressions hold even under the full SU(3) symmetry [3].

There are four $B \rightarrow KK\bar{K}$ decays in which the final KK pair is in a symmetric isospin state. However, only the amplitudes of $B^0 \rightarrow K^+K^0K^-$ and $B^0 \rightarrow K^0K^0\bar{K}^0$ are independent [2]. These are

$$\begin{aligned} \sqrt{2}A(B^0 \rightarrow K^+K^0K^-)_{sym} &= -T'_2 e^{i\gamma} - C'_1 e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} + \tilde{P}'_{tc} \\ &\quad + \kappa\left(\frac{1}{3}T'_1 - \frac{1}{3}C'_1 + \frac{2}{3}C'_2\right) , \\ A(B^0 \rightarrow K^0K^0\bar{K}^0)_{sym} &= \tilde{P}'_{uc} e^{i\gamma} - \tilde{P}'_{tc} + \kappa\left(\frac{2}{3}T'_1 + \frac{1}{3}C'_1 + \frac{1}{3}C'_2\right) . \end{aligned} \quad (10)$$

Note: since SU(3) has been assumed, $B \rightarrow KK\bar{K}$ diagrams in which the popped quark pair is $s\bar{s}$ are equivalent to $B \rightarrow K\pi\pi$ diagrams with a popped $u\bar{u}$ or $d\bar{d}$. This implies that $A(B^+ \rightarrow K^+\pi^+\pi^-)_{sym} = A(B^0 \rightarrow K^+K^0K^-)_{sym}$.

It is straightforward to show that one can combine the diagrams into “effective diagrams” $T'_a, T'_b, P'_a, P'_b, C'_a$ [4], giving

$$\begin{aligned} 2A(B^0 \rightarrow K^+\pi^0\pi^-)_{sym} &= T'_a e^{i\gamma} + T'_b e^{i\gamma} - C'_a - \kappa T'_b , \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^+\pi^-)_{sym} &= -T'_a e^{i\gamma} - P'_a e^{i\gamma} + P'_b , \\ \sqrt{2}A(B^0 \rightarrow K^+K^0K^-)_{sym} &= -P'_a e^{i\gamma} + P'_b - C'_a , \\ A(B^0 \rightarrow K^0K^0\bar{K}^0)_{sym} &= P'_a e^{i\gamma} - T'_b e^{i\gamma} - \frac{1}{\kappa}C'_a e^{i\gamma} - P'_b + \kappa T'_a + \kappa T'_b + C'_a . \end{aligned} \quad (11)$$

The 5 effective diagrams involve 10 unknown theoretical parameters: 5 magnitudes of diagrams, 4 relative strong phases, and γ . But there are 11 (momentum-dependent) experimental observables: the decay rates and direct asymmetries for $B^0 \rightarrow K^+\pi^0\pi^-$, $B^0 \rightarrow K^0\pi^+\pi^-$, $B^0 \rightarrow K^+K^0K^-$ and $B^0 \rightarrow K^0K^0\bar{K}^0$, and the indirect asymmetries of $B^0 \rightarrow K^0\pi^+\pi^-$, $B^0 \rightarrow K^+K^0K^-$ and $B^0 \rightarrow K^0K^0\bar{K}^0$. *With more observables than theoretical parameters, γ can be extracted from a fit.* Furthermore, because the observables and diagrams are momentum dependent, this analysis applies to every point in the Dalitz plot. Thus, this method actually constitutes many independent measurements of γ ! These can be averaged, reducing the error.

The above is a broad overview of the $K\pi\pi/KK\bar{K}$ method of measuring γ . However, using the experimental Dalitz-plot data from BaBar [6], my collaborators and I are in the process of carrying out this analysis. Details are given in the talk by B. Bhattacharya [7], but here is a summary. Only 14 points in the Dalitz plots have been used in this preliminary analysis, SU(3) breaking has not been taken into account, not all sources of error have been included, and there are multiple overlapping solutions (discrete ambiguities). With these caveats, the initial result is

$$\gamma = \left(81_{-5}^{+4} \text{ (avg.)} \pm 4 \text{ (std. dev.)}\right)^\circ. \quad (12)$$

This is consistent with independent direct measurements of γ . The Particle Data Group gives $\gamma = \left(66_{-10}^{+11}\right)^\circ$ [8].

Though preliminary, the result is extremely encouraging. It does indeed appear that one can cleanly extract weak-phase information from 3-body B decays, contrary to what was previously thought. Furthermore, there are indications that these measurements might be of high precision.

Of course, our analysis is based only on published data. I therefore strongly encourage the experimentalists to incorporate this method into their measurements of the 3-body $K\pi\pi/KK\bar{K}$ Dalitz plots. There is no doubt that many of the outstanding question marks could be better treated with a complete analysis of the experimental data, and the error on γ might be reduced even further.

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References

- [1] K. Abe *et al.* [Belle Collaboration], hep-ex/0208030; Y. Grossman, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D **68**, 015004 (2003) [hep-ph/0303171].
- [2] N. Rey-Le Lorier, M. Imbeault and D. London, Phys. Rev. D **84**, 034040 (2011) [arXiv:1011.4972 [hep-ph]].

- [3] N. Rey-Le Lorier, M. Imbeault and D. London, Phys. Rev. D **84**, 034041 (2011) [arXiv:1011.4973 [hep-ph]].
- [4] N. Rey-Le Lorier and D. London, Phys. Rev. D **85**, 016010 (2012) [arXiv:1109.0881 [hep-ph]].
- [5] M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998) [arXiv:hep-ph/9808493], Phys. Lett. B **441**, 403 (1998) [arXiv:hep-ph/9808493]; M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60**, 034021 (1999) [Erratum-ibid. D **69**, 119901 (2004)] [arXiv:hep-ph/9810482].
- [6] $B^0 \rightarrow K^+\pi^0\pi^-$: J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **83**, 112010 (2011) [arXiv:1105.0125 [hep-ex]]; $B^0 \rightarrow K^0\pi^+\pi^-$: B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **80**, 112001 (2009) [arXiv:0905.3615 [hep-ex]]; $B^0 \rightarrow K^+K^0K^-$: J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **85**, 112010 (2012) [arXiv:1201.5897 [hep-ex]]; $B^0 \rightarrow K^0K^0\overline{K}^0$: J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **85**, 054023 (2012) [arXiv:1111.3636 [hep-ex]].
- [7] B. Bhattacharya, D. London and M. Imbeault, these proceedings.
- [8] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).